# Gravitational Lensing Effect on the Two-point Correlation of Hotspots in the Cosmic Microwave Background

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## ABSTRACT

We investigate the weak gravitational lensing effect due to the large-scale structure of the universe on two-point correlations of local maxima (*hotspots*) in the 2D sky map of the cosmic microwave background (CMB) anisotropy. According to the Gaussian random statistics as most inflationary scenarios predict, the hotspots are discretely distributed with some *characteristic* angular separations on the last scattering surface owing to oscillations of the CMB angular power spectrum. The weak lensing then causes pairs of hotspots which are separated with the characteristic scale to be observed with various separations. We found that the lensing fairly smoothes the oscillatory features of the two-point correlation function of hotspots. This indicates that the hotspots correlations can be a new statistical tool for measuring shape and normalization of the power spectrum of matter fluctuations from the lensing signatures.

*Subject headings:* cosmology:theory – cosmic microwave background – gravitational lensing – large-scale structure of universe

## 1. Introduction

The temperature anisotropy of the cosmic microwave background (CMB) is the most powerful probe of our universe because the anisotropies encode a wealth of cosmological information and the physical processes involved in anisotropy formation are well understood by linear perturbation theory (e.g., Hu, Sugiyama & Silk 1997). Therefore, high-precision measurements of the primary CMB anisotropies enable us to determine a lot of fundamental cosmological parameters with unprecedented precision (Jungman et al. 1996; Bond, Efstathiou & Tegmark 1997; Zaldarriaga, Spergel & Seljak 1997). On the other hand, gravitational lensing due to the large-scale structure between the last scattering surface (LSS) and us deforms the 2D sky map of the CMB anisotropy. The weak lensing will be in the near future an effective tool for mapping the inhomogeneous distributions of dark matter because it is not affected by uncertainties in the "biasing" problem regarding the extent to which luminous objects trace the mass distribution. Furthermore, a great advantage in studying the lensing effect on the CMB anisotropies is that statistical properties of the unlensed CMB field can be fully specified once we assume the field obeys the Gaussian random theory (Bardeen et al. 1986, hereafter BBKS; Bond & Efstathiou 1987 hereafter BE). Most inflationary scenarios indeed support this assumption. However, most previous studies have focused on theoretical predictions of lensed shapes of the two-point correlation function $C(\theta)$ or equivalently the angular power spectrum $C_l$ of the temperature fluctuations field (Blanchard & Schneider 1987; Cole &

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Efstathiou 1989; Sasaki 1989; Tomita & Watanabe 1989; Cayón, Martńez-González & Sanz 1993a, b; Seljak 1996; Metcalf & Silk 1997; Zaldarriaga & Seljak 1998), while some studies have shown that the lensing induces non-Gaussian signatures (Bernardeau 1997; Zaldarriaga 1999). It is therefore worth exploring an other statistical CMB estimator sensitive to the lensing signatures using more directly the advantage of predictability of the 2D CMB sky map.

Recently, Heavens and Sheth (1999, hereafter HS) investigated the two-point correlation function of local maxima (*hotspots*) above a certain threshold in the 2D CMB map, say $\xi_{\mathrm{pk-pk}}(\theta)$, which can be accurately calculated based on the Gaussian random theory once $C_l$ is given. Their purpose was to propose that $\xi_{\mathrm{pk-pk}}(\theta)$ can be a powerful test of the Gaussian hypothesis of the temperature fluctuations, and they showed that the shape of $\xi_{\mathrm{pk-pk}}(\theta)$ is largely different from that of $C(\theta)$. $\xi_{\mathrm{pk-pk}}(\theta)$ has oscillatory features such as the depression around the angular scale corresponding to the 1st Doppler peak of $C_l$ and a prominent peak with a damping tail toward the smaller scales than the peak-scale. The reasons can be explained as follows. In the derivation of $\xi_{\mathrm{pk-pk}}(\theta)$, we need statistical properties of the gradient and second derivative fields of the temperature fluctuations field in order to identify the local maxima in the 2D CMB map. The shape of $\xi_{\mathrm{pk-pk}}(\theta)$ thus reflects more strongly the oscillatory features of $C_l$ such as a series of Doppler peaks than $C(\theta)$ does, because the gradient and the second derivative fields are more sensitive to the oscillations of $C_l$. The oscillations of $\xi_{\mathrm{pk-pk}}(\theta)$ physically mean that the hotspots are discretely distributed with some *characteristic* angular separations on the LSS. Then, the weak lensing due to the large-scale structure causes the pairs of hotspots separated with the characteristic scale on the LSS to be observed with various separations in the random line of sight. This leads to the expectation that $\xi_{\mathrm{pk-pk}}(\theta)$ enables us to see more directly the lensing dispersion of the specific angular separation than using $C(\theta)$. The purpose of this Letter is therefore to develop the formula for calculating the lensing effect on $\xi_{\mathrm{pk-pk}}(\theta)$ using the power spectrum approach to compute the dispersion of the lensing deflection angle (Seljak 1996), and to present some results in the standard cold dark matter (SCDM) model.

## 2. Weak Lensing Effect on the Hotspots Correlation Function

During the propagation from the LSS to us, a CMB photon is randomly deflected by the inhomogeneous matter distributions in the intervening large-scale structure. Then, two CMB photons observed with a certain angular separation $\theta$ have a different separation when emitted on the LSS. If we assume that the lensing fluctuations of the relative angular separation, $\delta\boldsymbol{\theta}(=\delta\boldsymbol{\theta}_1-\delta\boldsymbol{\theta}_2)$, obey the Gaussian random statistics (Seljak 1996), the ensemble average of the following characteristic function can be calculated as

$$\langle\exp[i\boldsymbol{l}\cdot(\delta\boldsymbol{\theta}_1-\delta\boldsymbol{\theta}_2)]\rangle_{|\boldsymbol{\theta}_1-\boldsymbol{\theta}_2|=\theta}\simeq\exp\left[-\frac{l^2}{2}\sigma^2_{\mathrm{GL}}(\theta)\right], \tag{1}$$

where $\delta\boldsymbol{\theta}_1$ and $\delta\boldsymbol{\theta}_2$ are the angular excursions of the two photons, and $\langle\ \rangle_\theta$ denotes the average performed over all pairs with a fixed observed angular separation $\theta$. The $\sigma^2_{\mathrm{GL}}(\theta)$ is defined by

$$\sigma^2_{\mathrm{GL}}(\theta)=\frac{9}{2\pi}H_0^4\Omega_{\mathrm{m0}}^2\int_0^\infty\frac{dk}{k}\int_0^{\chi_{\mathrm{rec}}}d\chi a^{-2}P_\delta(k,\tau)W^2(\chi,\chi_{\mathrm{rec}})(1-J_0(\chi k\theta)), \tag{2}$$

where $\tau$ is a conformal time, $\chi\equiv\tau_0-\tau$, $J_n(x)$ is the Bessel function of order $n$, and the subscript 0 and "rec" denote values at present and a recombination time, respectively. $P_\delta(k,\tau)$ is the power spectrum of the matter fluctuations, $H_0=100h$ km s$^{-1}$ Mpc$^{-1}$ and $\Omega_{\mathrm{m0}}$ denote the present Hubble parameter and the present energy density of matter, respectively. The projection operator $W(\chi,\chi_{\mathrm{rec}})$ on the celestial sphere is

given by $W(\chi, \chi_{\rm rec}) = 1 - \chi/\chi_{\rm rec}$ in a flat universe. The approximation used in the derivation of equation (1) is valid as long as the $\sigma_{\rm GL}(\theta)/\theta \ll 1$ is satisfied, and we numerically confirmed $\sigma_{\rm GL}/\theta < 0.25$ on the relevant angular scales for all normalizations of matter power spectrum considered in this Letter.

The unlensed $C(\theta)$ in the context of the small angle approximation developed by BE is given by $C(\theta) = \langle \Delta(\boldsymbol{\theta}_1)\Delta(\boldsymbol{\theta}_2)\rangle = \int (ldl/2\pi)S(l)J_0(l\theta)$, where $|\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2| = \theta$ and $\Delta(\boldsymbol{\theta}) \equiv (T(\boldsymbol{\theta}) - T_{\rm CMB})/T_{\rm CMB}$. The 2D power spectrum of the temperature fluctuations field, $S(l)$, is identical to $C_l$ in the limit of $l \gg 1$.

First, we briefly present the derivation of the unlensed two-point correlation function of hotspots in the 2D Gaussian random CMB field developed by HS. The moments of power spectrum are defined by $\sigma_i^2 \equiv \int_0^\infty (ldl/2\pi)S(l)l^{2i}$. At a hotspot, the gradients of temperature fluctuations field, $\Delta_i \equiv \partial\Delta/\partial\theta^i$, vanish and the eigenvalues of the second derivative matrix, $\Delta_{ij} \equiv \partial^2\Delta/\partial\theta^i\partial\theta^j$, are negative. We thus need 6 independent variables $\boldsymbol{v} = (\Delta, \Delta_x, \Delta_y, \Delta_{xx}, \Delta_{xy}, \Delta_{yy})$ to specify one local hotspot. The probability density function of the 12 variables $\boldsymbol{v} \equiv (\boldsymbol{v}_{(1)}, \boldsymbol{v}_{(2)}) = (\boldsymbol{v}(\boldsymbol{\theta}_1), \boldsymbol{v}(\boldsymbol{\theta}_2))$ for the two hotspots separated with the angle $\theta(= |\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2|)$ is defined by

$$p_2(\boldsymbol{v}_{(1)}, \boldsymbol{v}_{(2)}) = \frac{1}{(2\pi)^6|M|^{1/2}} \exp\left[-\frac{1}{2}v_i M_{ij}^{-1} v_j\right], \tag{3}$$

where $M_{ij}$ is the covariance matrix: $M_{ij} \equiv \langle (v_i - \langle v_i\rangle)(v_j - \langle v_j\rangle)\rangle$, $M = \det(M_{ij})$, and $M_{ij}^{-1}$ is the inverse of $M_{ij}$. Note that $\langle v_i\rangle = 0$ in the present case. It is then convenient to introduce the variables defined by $\nu \equiv \Delta/\sigma_0, \eta_i \equiv \Delta_i/\sigma_1, X \equiv -(\Delta_{xx} + \Delta_{yy})/\sigma_2, Y \equiv (\Delta_{xx} - \Delta_{yy})/\sigma_2$, and $Z \equiv 2\Delta_{xy}/\sigma_2$. All non-zero components of $M_{ij}$ for two hotspots characterized by $\nu_{(1)} \equiv \nu(\boldsymbol{\theta}_1)$ and $\nu_{(2)} \equiv \nu(\boldsymbol{\theta}_2)$ and so on were explicitly calculated by HS. Employing the BBKS formalism, we can obtain the following unlensed correlation function of two hotspots above certain thresholds $\nu_1$ and $\nu_2$, respectively:

$$\begin{aligned} 1 + \xi_{\rm pk-pk}(\theta| > \nu_1, > \nu_2) \quad \equiv \quad & \frac{1}{\bar{n}_{\rm pk}(>\nu_1)\bar{n}_{\rm pk}(>\nu_2)} \langle n_{\rm pk}(\boldsymbol{\theta}_1) n_{\rm pk}(\boldsymbol{\theta}_2)\rangle \\ = \frac{1}{2^2\theta_*^4 \bar{n}_{\rm pk}(>\nu_1)\bar{n}_{\rm pk}(>\nu_2)} & \int_{\nu_1}^\infty d\nu'_{(1)} \int_{\nu_2}^\infty d\nu'_{(2)} \int_0^\infty dX_{(1)} \int_0^\infty dX_{(2)} \int_{-X_{(1)}}^{X_{(1)}} dY_{(1)} \int_{-X_{(2)}}^{X_{(2)}} dY_{(2)} \\ \times & \int_{-\sqrt{X_{(1)}^2 - Y_{(1)}^2}}^{\sqrt{X_{(1)}^2 - Y_{(1)}^2}} dZ_{(1)} \int_{-\sqrt{X_{(2)}^2 - Y_{(2)}^2}}^{\sqrt{X_{(2)}^2 - Y_{(2)}^2}} dZ_{(2)} (X_{(1)}^2 - Y_{(1)}^2 - Z_{(1)}^2)(X_{(2)}^2 - Y_{(2)}^2 - Z_{(2)}^2) \\ \times & \; p_2(\nu'_{(1)}, X_{(1)}, Y_{(1)}, Z_{(1)}, \eta_{(1)i} = 0, \nu'_{(2)}, X_{(2)}, Y_{(2)}, Z_{(2)}, \eta_{(2)i} = 0), \end{aligned} \tag{4}$$

where $\theta_* = \sqrt{2}\sigma_1/\sigma_2$, $n_{\rm pk}(\boldsymbol{\theta})$ and $\bar{n}_{\rm pk}(>\nu)$ are the number density filed and the mean number density of hotspots above height $\nu$, respectively. To obtain $\xi_{\rm pk-pk}(\theta)$, we performed a 6D numerical integration of equation (4) because the two can be done analytically.

Next, we derive the lensed correlation function of hotspots. In this Letter, we focus on the lensing contributions to $\xi_{\rm pk-pk}(\theta)$ as a secondary effect which comes from a mapping of the intrinsic discrete distributions of hotspots on the LSS. The procedure is appropriate because a power of anisotropies generated by the lensing is small (Seljak & Zaldarriaga 1999; Zaldarriaga 1999). For this purpose, we define the fluctuations field of $n_{\rm pk}(\boldsymbol{\theta})$ as $\delta n(\boldsymbol{\theta}) \equiv (n_{\rm pk}(\boldsymbol{\theta}) - \bar{n}_{\rm pk})/\bar{n}_{\rm pk}$. Likewise, since the gravitational lensing makes us to observe, at a certain angular position $\boldsymbol{\theta}$, a hotspot at a different position $\boldsymbol{\theta} + \delta\boldsymbol{\theta}$ on the LSS, the lensed (observed) fluctuations field $\delta\tilde{n}(\boldsymbol{\theta})$ can be expressed as $\delta\tilde{n}(\boldsymbol{\theta}) \equiv \delta n(\boldsymbol{\theta} + \delta\boldsymbol{\theta}) = \int d^2\boldsymbol{l}/(2\pi)^2 \delta n(\boldsymbol{l}) \exp[i\boldsymbol{l}\cdot(\boldsymbol{\theta} + \delta\boldsymbol{\theta})]$, where $\delta n(\boldsymbol{l})$ is the Fourier component of $\delta n(\boldsymbol{\theta})$. If assuming that the lensing fluctuations and the intrinsic CMB anisotropies field are statistically independent, then the lensed two-point correlation function of

hotspots can be calculated with help of equation (1) as

$$\xi^{\rm GL}_{\rm pk-pk}(\theta) \equiv \langle \delta\tilde{n}(\boldsymbol{\theta}_1)\delta\tilde{n}(\boldsymbol{\theta}_2)\rangle \simeq \frac{1}{\sigma^2_{\rm GL}(\theta)}\int d\theta'\theta'\xi_{\rm pk-pk}(\theta')\exp\left[-\frac{\theta^2+\theta'^2}{2\sigma^2_{\rm GL}(\theta)}\right] I_0\left(\frac{\theta\theta'}{\sigma^2_{\rm GL}(\theta)}\right). \qquad (5)$$

where $I_0(x)$ is the modified zeroth-order Bessel function. This equation indicates that the lensing contribution at a certain scale $\theta$ arises only from the lensing dispersion $\sigma_{\rm GL}(\theta)$ at the same scale alone. Hence detecting the scale dependence of the lensing contributions to $\xi_{\rm pk-pk}(\theta)$ enables us to reconstruct the lensing dispersion, more importantly the projected matter power spectrum by equation (2).

## 3. Results

In the following, we show some results in the SCDM model with $\Omega_{\rm m0} = 1$. The Hubble parameter and the present baryon density are $h = 0.6$ and $\Omega_{\rm b0}h^2 = 0.015$, respectively. To compute the intrinsic CMB angular power spectrum, $S(l)$, we used the CMBFAST code developed by Seljak & Zaldarriaga (1996). As for the matter power spectrum in the lensing dispersion (2), we employed the Harrison-Zel'dovich spectrum and the BBKS transfer function with the shape parameter in Sugiyama (1995). A free parameter in our model is only the normalization of the present-day matter power spectrum which is conventionally expressed in terms of the rms mass fluctuations within a sphere of $8h^{-1}$Mpc, i.e., $\sigma_8$.

In Fig. 1, we show the two-pint correlation function of hotspots of height above the threshold $\nu = 1$ with and without gravitational lensing effect, $\xi^{\rm GL}_{\rm pk-pk}(\theta)$ and $\xi_{\rm pk-pk}(\theta)$, respectively. Note that the intrinsic $\xi_{\rm pk-pk}(\theta)$ can be calculated by using only $C_l$. The mean number density of hotspots is then $\bar{n}_{\rm pk}(>\nu) = 1.16\times10^4{\rm rad}^{-2}$. The dependence of the lensing effect on the normalization of mass fluctuations is demonstrated by the four choices of $\sigma_8 = 0.5, 1, 1.5, 2$, where $\sigma_8 = 0.5$ and 1 roughly correspond to normalizations of the cluster abundance (Eke, Cole & Frenk 1996; Kitayama & Suto 1997) and the COBE measurements (Bunn & White 1997), respectively. The figure clearly shows that $\xi_{\rm pk-pk}(\theta)$ has more oscillatory features than $C(\theta)$. This is because the gradient and second derivative fields of $\Delta(\boldsymbol{\theta})$ used in the derivation of $\xi_{\rm pk-pk}(\theta)$ have power spectra of $C_l l^4/(2\pi)$ and $C_l l^6/(2\pi)$ per logarithmic interval in $l$, respectively, and thus they strongly enhance the oscillations of $l^2C_l/(2\pi)$, while $C(\theta)$ is fully determined by $l^2C_l/(2\pi)$. If using the relation $l \approx \pi/\theta$, there are indeed correspondences between both oscillations of $\xi_{\rm pk-pk}(\theta)$ and $l^2C_l$ such as the depression at $\theta \approx 75'$ corresponding to the scales around the 1st Doppler peak of $C_l$, a prominent peak at $\theta \approx 13'$, and a damping tail at $\theta < 10'$ associated with the Silk damping. Moreover, Fig. 1 shows that the gravitational lensing appears as a larger smoothing at scales where $\xi_{\rm pk-pk}(\theta)$ has more oscillatory features, since pairs of hotspots separated with the characteristic angular scales corresponding to the oscillations of $\xi_{\rm pk-pk}$ are redistributed by the weak lensing and thus power of $\xi_{\rm pk-pk}(\theta)$ is transfered from (or to) the scale to (or from) nearby scales. Especially, it should be stressed that even the lensing effect on the depression at $\theta \approx 75'$ associated with the 1st Doppler peak of $C_l$ is distinguishable where the deviation $\delta\xi$ $(\equiv (\xi^{\rm GL}_{\rm pk-pk} - \xi_{\rm pk-pk})/\xi_{\rm pk-pk})$ reaches up to $\approx 20\%$ for $\sigma_8 = 2.0$ while $\delta C < 1.0\%$ in all cases (see Fig. 2). Since both $\xi^{\rm GL}_{\rm pk-pk}$ and $\xi_{\rm pk-pk}$ give the same results as the shape becomes flat on larger scales ($\theta > 80'$), the large scale amplitude of $\xi_{\rm pk-pk}$ gives the normalization independent of the lensing contributions. We also demonstrate the nonlinear corrections to the matter power spectrum to clarify how the nonlinear evolution of matter fluctuations affects our analysis. Using the semi-analytic formulae developed by Peacock & Dodds (1996) for the extreme case of $\sigma_8 = 2$, where the universe has too large power of the present-day small-scale matter fluctuations, we found that the nonlinear correction is small at $\theta > 5'$. Most importantly, Fig. 1 indicates that the magnitude of the lensing contributions to $\xi_{\rm pk-pk}$ is fairly sensitive to the amplitude of $\sigma_8$. Furthermore, measuring scale-dependence

of the lensing contributions such as $\delta\xi(\theta)$ at $\theta = 2', 16', 36', 44', 75'$ will allow us to reconstruct the shape of projected power spectrum of matter fluctuations in the range of $3h^{-1}\mathrm{Mpc} \lesssim \lambda \lesssim 50h^{-1}\mathrm{Mpc}$.

## 4. Discussions and Conclusions

In this Letter, we have investigated the weak lensing effect on the two-point correlation of hotspots in the CMB. One of the great advantages in studying the lensing effect on the CMB anisotropies is that statistical properties of unlensed CMB field are fully predictable by the Gaussian random theory. For example, Bernardeau (1998) have investigated how the lensing changes probability distributions of the ellipticity defined from the local temperature curvature matrix, and shown that the method can be a statistical indicator sensitive to the lensing signatures.

It is now expected that the weak lensing can be a powerful probe for measuring the power spectrum of matter fluctuations. In case of the normalization based on measurements of the large angular scale CMB anisotropies such as COBE normalization (e.g., Bunn & White 1997), we need to employ the very accurate matter transfer function which highly depends on a lot of cosmological parameters in order to determine the amplitude of small scale matter fluctuations. In fact, even most sensitive future experiments such as the *Planck Surveyor* (Bersanelli et al. 1996) will constrain $\sigma_8$ up to only 20% accuracy (see, e.g., Bond, Efstathiou & Tegmark 1997). We thus wish to propose that $\xi_{\mathrm{pk-pk}}(\theta)$ can be a new effective statistical tool for measuring amplitude and shape of the matter power spectrum.

For this purpose, we have to perform quantitative predictions whether or not the future satellite missions, *MAP* (Bennet et al. 1996) and *Planck*, can measure the lensing contributions to $\xi_{\mathrm{pk-pk}}(\theta)$ and constrain $\sigma_8$ with high accuracy. In particular, we have to estimate not only uncertainties by the cosmic variance but also observational errors in identifying angular positions of hotspots in a realistic 2D sky map of the CMB anisotropies. It is then important to take into account the following experimental effects. (1) The finite beam size of detectors may incorporate some hotspots within the beam size resulting from smoothing of the lensed CMB anisotropies filed. The effect hence decreases the mean number density of hotspots and suppresses two-point correlations of hotspots on scales below the beam size. (2) The detector noise may make spurious hotspots in the observed CMB map. HS estimated the errors for measurements of the unlensed hotspots correlation functions expected from *MAP* and *Planck* by using numerical sky realizations, and indeed showed that the theoretical predictions including the above experimental effects are in good agreement with the numerical results. Following their method, we are now investigating the detectability of the lensing signatures to $\xi_{\mathrm{pk-pk}}(\theta)$, and have obtained the preliminary result as follows. Supposing the data produced by the *Planck* specification with 65% sky coverage, the expected signal to noise ratios for the lensing signatures are $S/N \approx 1.3, 1.8, 1.5$ and $3.1$ at $\theta = 16', 36', 44'$ and $75'$, respectively, for $\sigma_8 = 1.5$ model. Furthermore, we expect that the statistical significance of $S/N$ could be increased by combining measurements of the two-point correlations of *coldspots*. The observed correlations of coldspots should be identical to that of hotspots since they are subject to the Gaussian random statistics. These works are now in progress and will be presented in detail elsewhere.

We finally comment on possibilities that the lensing effect on $\xi_{\mathrm{pk-pk}}$ breaks the so-called *geometrical degeneracy* (Bond, Efstathiou & Tegmark 1997; Metcalf & Silk 1998; Stompor & Efstathiou 1999) inherent in cosmological parameter determinations from measurements of the CMB anisotropies. We have confirmed that the global shape of $\xi_{\mathrm{pk-pk}}(\theta)$ is sensitive to fundamental cosmological parameters such as $h, \Omega_{\mathrm{b}}, \Omega_{\mathrm{m0}}$ and so on. We thus expect that detecting $\theta$-dependence of the lensing contributions to $\xi^{\mathrm{GL}}_{\mathrm{pk-pk}}(\theta)$ can yield

an additional constraint on the $\sigma_8 - \Omega_{\rm m0}$ plane if we assume the shape of matter power spectrum. Hence, the degeneracy could be broken by observations of the CMB anisotropies alone, without help of other astronomical measurements.

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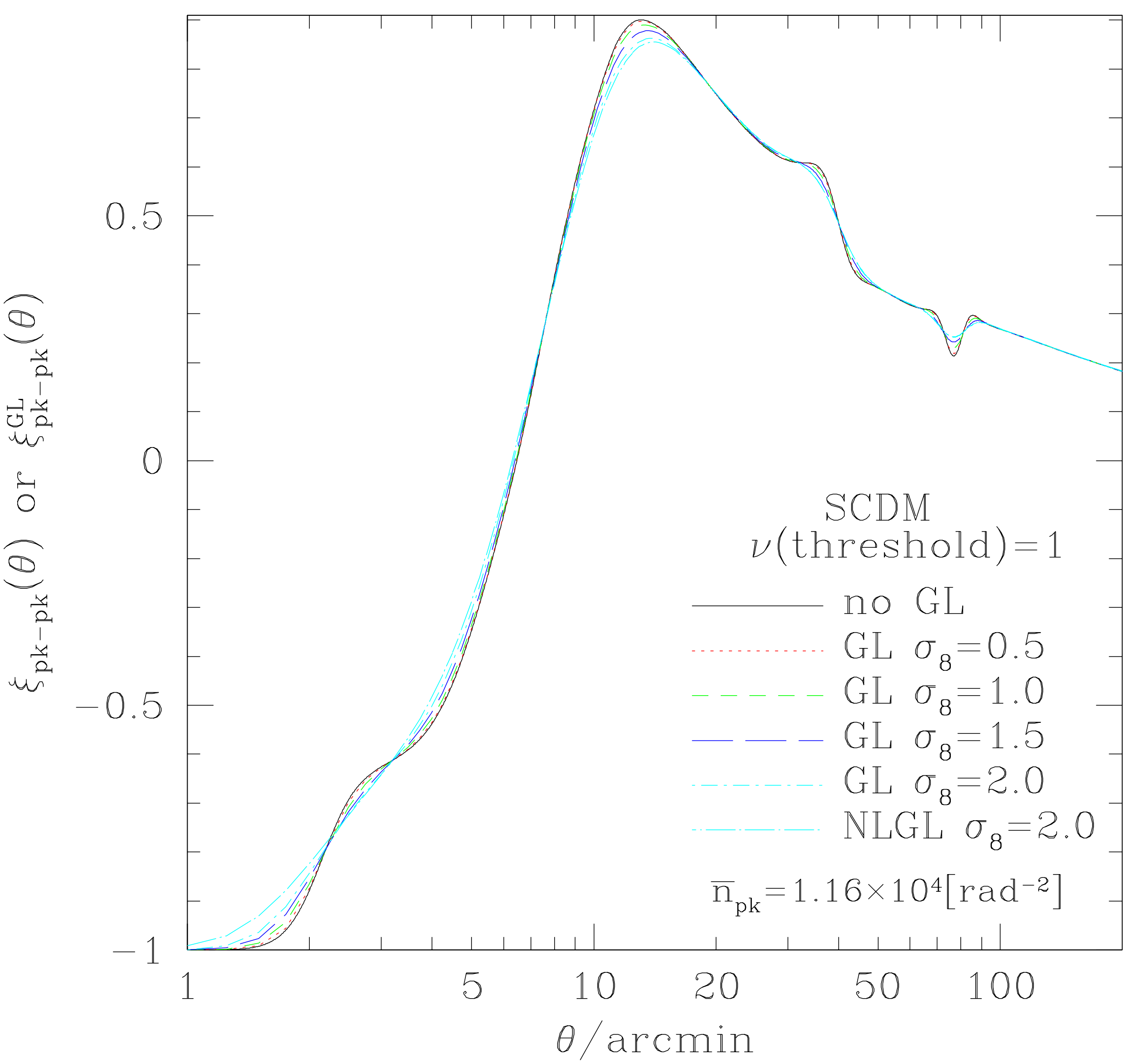


Fig. 1.— The correlation function of hotspots vs. $\theta$ with and without the gravitational lensing effect in SCDM model. The solid line is the unlensed correlation function, while dotted, short-dashed, long-dashed and dotted-dashed lines represent the corresponding lensing cases with $\sigma_8 = 0.5, 1.0, 1.5$ and $2.0$, respectively. The long-dotted-dashed line denotes the result including nonlinear correction with $\sigma_8 = 2.0$ (see text). $h = 0.6$ and $\Omega_{\rm b0}h^2 = 0.015$ are assumed. The threshold of hotspots is $\nu = 1$. The mean density of hotspots is then $\bar{n}_{\rm pk} = 1.16 \times 10^4 {\rm rad}^{-2}$.

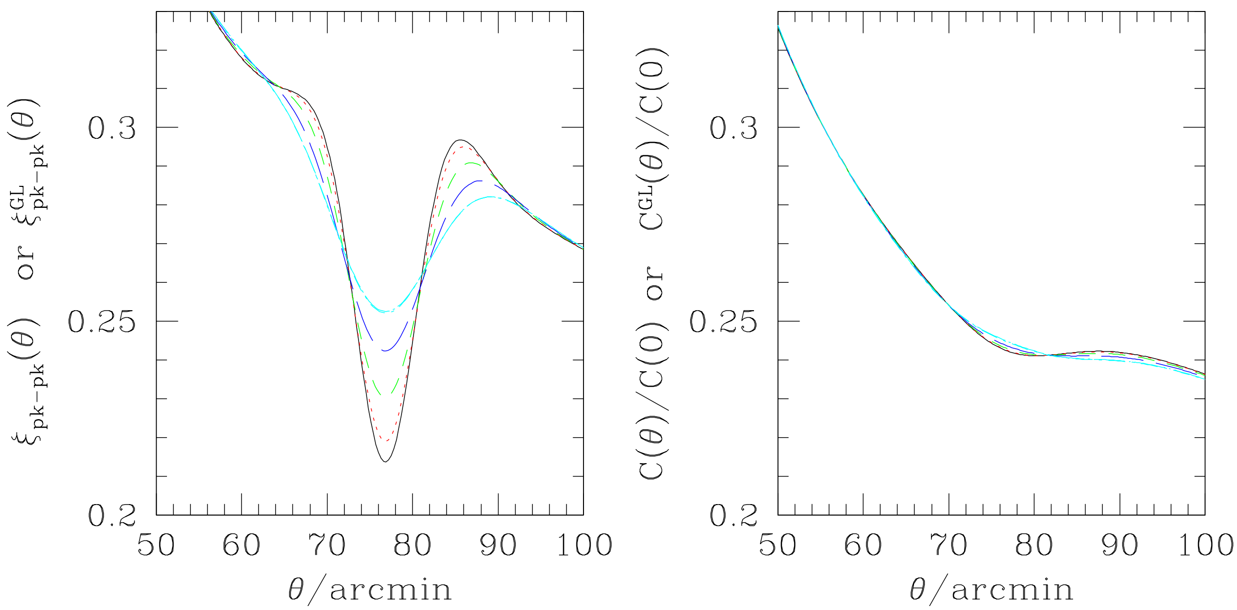


Fig. 2.— The two-point correlation functions of hotspots and the temperature anisotropies field around the 1st Doppler peak with and without the gravitational lensing effect.